E 2234 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, AUGUST 2021

Third Semester

Complementary Course—Statistics

PROBABILITY DISTRIBUTIONS

[Common for B.Sc. Mathematics, Physics and Computer Applications [Three Main]]

(2013—2016 Admissions)

Time : Three Hours Maximum Marks : 80

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Write the p.m.f. of the binomial distribution with parameters n and p.
2. Write the moment generating function of a Poisson distribution with parameter λ.
3. Obtain the mean of geometric distribution.
4. Define standard normal distribution.
5. Write the p.d.f. of uniform distribution over the interval (a, b).
6. Write the p.d.f. of the exponential distribution with mean θ.
7. State the additive property of normal distribution.
8. State Lindberg-Levy form of Central Limit theorem.
9. Define Statistic.
10. Write the p.d.f. of Chi-square distribution with n degrees of freedom.

$(10 \times 1 = 10)$

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Show that geometric distribution possesses lack of memory property.
12. Obtain the mean and variance of a Poisson distribution with parameter λ.
13. State and prove the additive property of the binomial distribution.
14. Find the mean of Beta distribution of the second kind.
15. Derive the mean of the normal distribution, $N(\mu, \sigma^2)$.

Turn over

16. What are the properties of Normal distribution ?

17. Derive the mean of the gamma distribution with p.d.f.

$$f(x) = \frac{m^p}{\Gamma(p)} e^{-mx} x^{p-1}, \quad x > 0$$
$$= 0, \quad \text{otherwise}$$

18. State memorylessness property of exponential distribution.

19. Find the least value of probability $p\{1 \le X \le 7\}$, where X is a r.v with E (X) = 4 and Var (X) = 4.

20. State Bernoulli's law of large numbers.

21. Define Sampling distributions.

22. Define t-statistic.

(8 × 2 = 16)

Part C (Description/Short Essays)

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Obtain modes of a binomial distribution with parameters n and p.

24. Derive the moment generating function of a binomial distribution with parameters n and p. Hence find its mean and variance.

25. Derive mean deviation about the mean of a $N(\mu, \sigma^2)$ distribution.

26. Find the distribution function of a random variable with p.d.f. given by

$$f(x; a, b) = \frac{1}{b-a}, \quad \text{if } a < x < b$$
$$= 0, \quad \text{otherwise}$$

Also draw the graphs of p.d.f. and distribution function.

27. Derive the distribution of sample mean if samples are taken from $N(\mu, \sigma^2)$.

28. State and prove the additive property of the Chi-square distribution.

29. A random variable X has mean 50 and variance 100. Use Chebyshev's inequality to obtain appropriate bounds for

(a) $P\{|X - 50| \ge 15\}$ and

(b) $P\{|X - 50| < 20\}$.

30. Let $\{X_n, n \geq 1\}$ be a sequence of independent and identically distributed Poisson random variable with mean λ. Obtain the limiting distribution of $\sum_{i=1}^{n} X_i$ with the help of Central Limit theorem.

31. Derive the moment generating function of Chi-square distribution with n degrees of freedom.

(6 × 4 = 24)

Part D (Essays)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. Fit Poisson distribution for the following data and also find expected frequencies :

Number of mistakes per page	:	0	1	2	3	4	Total
Number of pages	:	109	65	22	3	1	200

33. (a) Derive the points of inflexion of $N(\mu, \sigma^2)$.

(b) Establish the recurrence relation between central moments of a binomial distribution.

34. (a) State and prove Chebycheff's inequality.

(b) Let $\{X_n\}$ be a sequence of mutually independent and identically distributed random variables with mean μ and finite variance σ^2. If $S_n = X_1 + X_2 + ... + X_n$. Prove that the law of large numbers does not hold the sequence $\{S_n\}$.

35. (a) Derive the mean of the F distribution with parameters n_1 and n_2.

(b) If X_1 and X_2 are independent Chi-square random variables each with one degrees of freedom. Find λ such that $P\{X_1 + X_2 > \lambda\} = \frac{1}{2}$.

(2 × 15 = 30)